Activity 14: Relativity

This Activity is an introduction to Einstein's theory of relativity. In particular we'll introduce the *special theory of relativity*, so called because it is a "special case" of a more general theory. The *general theory of relativity* is a much larger and more complex model of gravity and spacetime that Einstein developed later. This lesson will follow a historical sequence and so we will start with the special theory of relativity because that's where Einstein himself began.

The Speed of Light Puzzle

In 1865, the Scottish physicist **James Clerk Maxwell** accomplished something incredible: he unified all of electricity and magnetism into a single model called **electromagnetic theory**. It is due to Maxwell's work that we know that light is an electromagnetic wave, which is a fact we've made frequent use of in this course.

James Clerk Maxwell (1831 – 1879)

Incredibly, Maxwell was able to use his model to *predict* what the speed of light *ought* to be, and *his prediction matched* the already known measured values of the speed of light.

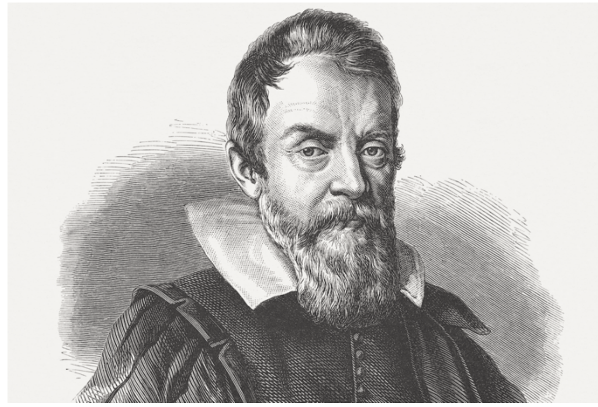
> Aside: The speed of light was first estimated by Danish astronomer Rømer in 1676 by studying the orbit of one of the moons of Jupiter (which is itself an amazing story for another lesson). Much later the speed of light was measured more directly in 1849 using a very clever spinning mirror apparatus invented by French physicists Fizeau and Foucault (the same Foucault of the famous eponymous pendulum). So at the time of Maxwell's work, the speed of light had already been both indirectly and directly measured.

Despite the immense success of Maxwell's description of light as an electromagnetic wave, there were still some open questions. The open question pertaining to today's topic is the problem of the speed of the light source and the speed of the observer.

Speed of Light Source/Observer Problem: Maxwell's model can successfully predict the speed of light based on electromagnetic laws, but this prediction doesn't utilize any information whatsoever about the motion of the light source or of the observer.

We'll first strive to understand why this is a "problem". Then, we will look at how Einstein solved this problem and was consequently led down the rabbit hole that led him to formulate his theory of relativity.

Part 1: Galilean Relativity



Galileo Galilei (1564-1642)

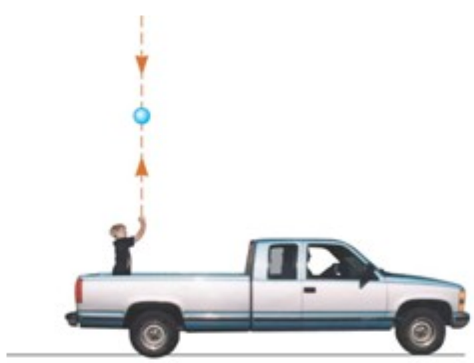
Relativity, at least in one form, was published by Galileo in 1632 (just 10 years before his death). So to begin our investigation of relativity we will first rewind to 1632.

STEP 1: Let's start with a thought experiment, a tool we introduced early in this course and one that was utilized frequently by both Galileo and Einstein.

Imagine you are riding in a car going down the highway at 60 mph. The windows are up so there is no air moving through the car. From the passenger seat, you toss a ball straight up in the air with enough force so that it doesn't quite reach the ceiling of the car. Where does the ball land? Does it land back in your hand? Does it fly toward the back of the car at 60 mph? Something else?

Would your previous answer change if you were going 75 mph instead of 60 mph?

Probably from your everyday experience riding in a car on the highway, you recall that something tossed upward doesn't fly backwards but instead comes straight back down. Imagine if this *weren't* the case and you are riding in the cabin of an airplane going 550 mph; anything tossed or dropped would become a deadly and destructive projectile!



What a person inside a moving car sees

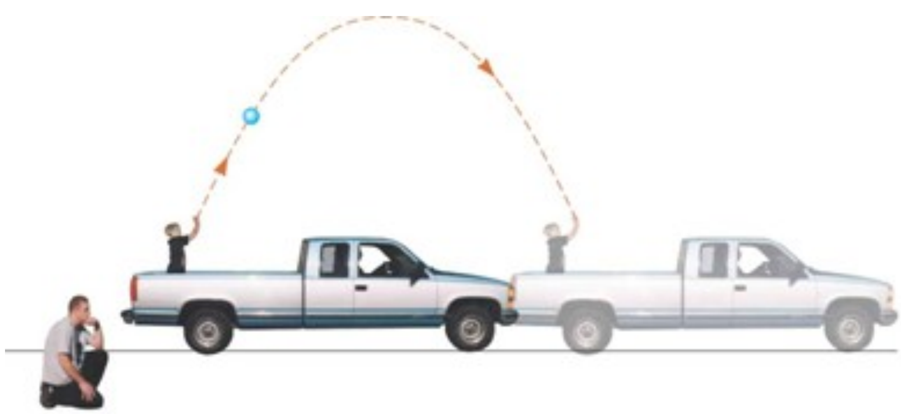
STEP 2: Now imagine the car's windows are all blacked out, so that you can't see anything outside the car. The car ride is smooth and noiseless. Without looking outside the car or using anything connected to the outside of the car (such as its speedometer), can you think of an experiment that you could perform that will tell you how fast the car is moving? If so, what would the experiment be?

If you thought of an experiment to determine the speed of the car without looking outside, share it with your instructor now.

If you came to the conclusion that no such experiment is possible, then you came to the same conclusion Galileo did.

In 1632, Galileo wrote that if a ship is sailing on a smooth sea (i.e., no rocking) at a constant speed, anyone below deck (where there are no windows) would not be able to tell whether the ship was moving or stationary. This argument is now known as "**Galileo's ship**".

STEP 3: Now, let's add another observer to the situation, one that is beside the road outside the car. If the car is moving, what path does the ball appear to take for the person outside the car? Consider the image below.



What a person outside a moving car sees

From this perspective, is the ball going up and coming back down? Is it also moving in any other direction?

In summary, when someone tosses a ball upward in a car, what path does the ball take?

A) Straight up and straight down
B) Upward at a diagonal and then downward at a diagonal
C) A parabolic shape (curved path)
D) It depends on who you ask!

Double check your previous answer with your instructor before continuing.

STEP 4: The key difference between the two people we've considered above is what we call their *reference frame*.

Reference Frame

A reference frame is the frame (i.e., environment, surroundings) in which an observer appears to be sitting still. Therefore, it is also often called the *rest frame*.

Reviewing the definition of reference frame above, what is your reference frame right now?

What is your reference frame when you are riding in a car?

What is your reference frame when you are standing by the side of the highway watching cars drive past you?

True or False: You are *always* at rest in your own reference frame. *Hint:* Look back at the definition.

STEP 5: Let's call the person inside the car (who's tossing the ball) "Person A". And the person watching from outside the car is "Person B".

SQ1:

In all of the following questions, we are talking about one and the same ball, but as seen from two different perspectives (the perspective of A versus the perspective of B).

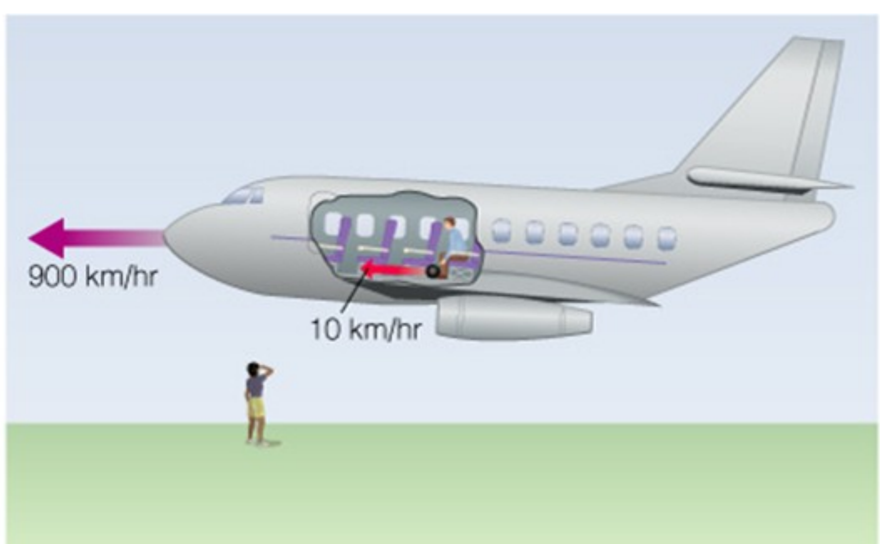
a) Do A and B agree or disagree on how far the ball travels? Explain how you can tell. And, if they disagree, who sees the ball travel farther? *Hint:* Look back at the drawings of the ball's path from each person's perspective.

b) Do A and B agree or disagree on how long the ball is airborne? In other words, if they both have stopwatches that they start when the ball leaves A's hand and stop when the ball returns to A's hand, would you expect their stopwatches would agree on how long the ball is airborne?

c) Do A and B agree or disagree on how fast the ball is traveling? If they disagree, who sees the ball traveling faster? *Hint:* speed equals distance per time.

STEP 6: Let's do a similar thought experiment, this time where the ball and vehicle are moving in the same direction.

Consider the drawing below in which a person inside a plane (call them Person A) is rolling a ball toward the front of the plane's cabin, while a second person (call them Person B) is watching from outside. Notice the speeds are labeled on the drawing.



What is the reference frame for person A?

What is the reference frame for person B?

Since they are in different reference frames, do you expect them to agree or disagree on the speed of the ball?

How fast is the ball moving in the reference frame of A, in km/hr?

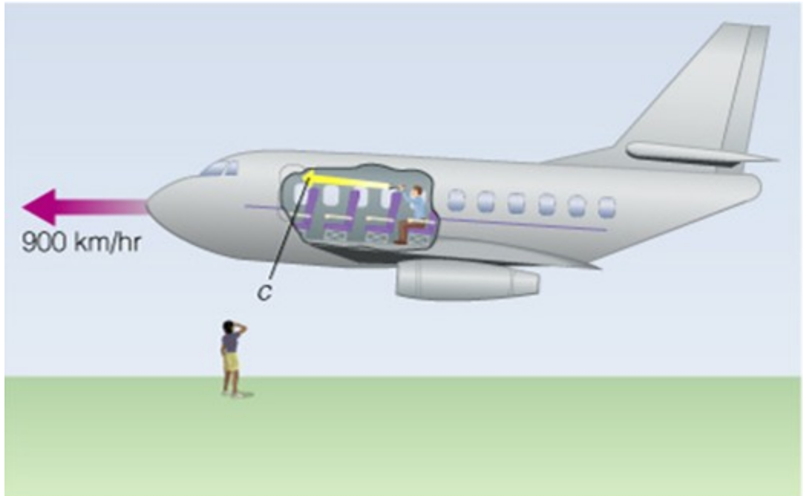
How fast is the ball moving in the reference frame of B, in km/hr? *Hint:* B sees the ball moving forward for *two* reasons: it is being rolled forward by A while A is on an airplane that is also moving forward.

SQ2: Let's summarize Galilean Relativity:

> Each observer is (at rest / moving) in their own reference frame. Two
>
> observers that are moving with respect to each other have different
>
> ______________________________ and therefore will (agree / disagree)
>
> on the speed at which something is moving.

STEP 7: Let's repeat the "ball on the plane" thought experiment, but this time replace the ball with a flashlight, as depicted in the drawing below (next page).

As is customary, the **speed of light is denoted with the letter *c***.



SQ3:

a) How fast is the beam of light moving for Person A, inside the plane?

b) How fast would you expect the beam of light to be moving from the perspective of Person B, outside the plane? Express your answer as an arithmetic quantity.

c) Review the "Speed of Light Source/Observer Problem" at the beginning of this Part. Why is it a problem that Maxwell's prediction of the speed of light does not utilize any information about the motion of the light source or the motion of the observer?

Part 2: Einsteinian Relativity

Because of the “Speed of Light Source/Observer Problem”, it was thought that Maxwell’s electromagnetic theory was incomplete. For decades the missing elements of the theory went undiscovered and the problem remained unsolved. Then, in 1905, along came a 26-year-old patent clerk named Albert Einstein that proposed a solution.

Albert Einstein in 1904

Einstein proposed: what if Maxwell’s theory is fine just the way it is? What if it’s actually true that **the speed of light is always *c*, *regardless of any motion of the observer or the source***? In other words, Einstein’s solution was to suggest that the “Problem” isn’t a problem at all, and that there’s nothing missing from Maxwell’s electromagnetic model of light.

The phrase in bold above we will call Einstein’s postulate of the constancy of the speed of light**,** which is repeated in the box below.

Einstein’s Postulate

The speed of light is always *c*, regardless of any motion of the observer or the source.

This is a radical suggestion because, if it were true, it would lead to some very strange results. Let’s take a look at a couple of obvious (but still strange) results first. Then we’ll look into some of the deeper consequences of Einstein’s Postulate.

STEP 1: Returning to the “flashlight on the plane” thought experiment, let’s do it again *assuming that Einstein’s postulate is true*.

Before answering the next two questions, review Einstein’s Postulate above.

If Einstein’s Postulate were true, how fast must the beam of light be moving for Person A, inside the plane?

If Einstein’s Postulate were true, how fast must the beam of light be moving for Person B, outside the plane?

Compare your previous answer to your answer in part (b) of SQ3. The apparent inconsistency should already be coming into view!

STEP 2: Let's take a more extreme example, this time of a spaceship that's moving at half the speed of light (0.5c).

Person A is the pilot of the spaceship and Person B is watching the spaceship through a telescope from a nearby moon.

SQ4: The pilot turns on the spaceship's headlights. Assume Einstein's postulate is true. How fast does the beam of light, produced by the headlights, appear to be moving…

a) … in Person A's reference frame?

b) … in Person B's reference frame?

c) If instead we are thinking in Galilean terms (i.e., without Einstein's Postulate) as we did in Part 1, what might we expect the speed of the beam to be according to Person B?

Does this seem strange yet? It should. And it's about to get even stranger!

<u>Light Clocks</u>

Let's continue with the assumption that Einstein's postulate is true and investigate some of the deeper consequences of the postulate.

We're going to do another thought experiment. This time we're going to build clocks that use light as a means of keeping time, as follows:

- A beam of light bounces back and forth between two mirrors.
- We'll *define* the time it takes for a single round trip of the beam of light to be *one unit of time.* In other words, one round trip of the light is one "tick" of the light clock.

We build two of these light clocks and we give one to Jack and one to Jill.

STEP 3: Open the simulation below that involves two light clocks.

Here is a Flash version:

http://galileoandeinstein.physics.virginia.edu/more_stuff/flashlets/lightclock.swf

Here is a non-Flash version:

http://galileoandeinstein.physics.virginia.edu/more_stuff/Applets/Lightclock/home.html

Check the box next to “Show Light Path” (if using Flash) or “Trace” (if using non-Flash).

STEP 4: Jack will stay at rest always. Jill can move, and her speed will be adjusted using the slider in the simulation.

To begin with, set Jill’s speed slider to zero, so that both Jack and Jill are at rest. Run the simulation.

What is the round trip time (one “tick”) for Jack’s light clock?

What is the round trip time (one “tick”) for Jill’s light clock?

Are the light clocks moving with respect to one another?

Do the two light clocks tick at the same rate (i.e., do they have the same round trip time)?

STEP 5: Reset the simulation and turn on the Light Path/Trace again if needed. Set Jill’s speed to 0.25c (one quarter the speed of light). Run the simulation again.

Jack’s clock will continue to “tick” until Jill’s clock has completed one “tick”. Jack is using his light clock to time how long it takes Jill’s clock to tick once.

The following questions are from Jack’s reference frame, where his clock is at rest and Jill’s clock is moving:

> Comparing the distance traveled by the light in the two light clocks, for which clock did the light travel *farther*? Jack’s (the stationary one), Jill’s (the moving one), or neither? *Hint:* Look at the length of the lines that trace out the light’s path in the simulation.
>
> For which clock did the light travel *faster*? Jack’s (the stationary one), Jill’s (the moving one), or neither? Recall that we’re assuming Einstein’s postulate is true.

Based on your two previous answers, which light clock would have a longer round trip time (and therefore take longer to "tick")? Why?

STEP 6: Now put yourself in Jill's shoes. Jill is moving along with her light clock. Imagine Jill and her light clock are on a train with no windows and the train is very quiet. The light clock going up and down is analogous to throwing a ball up and down. Does Jill have any way of determining whether she is moving or not? *Hint:* What did we learn from Galilean Relativity (Galileo's ship)?

From Jill's perspective *within her own reference frame*, how fast does Jill appear to be going? *Hint:* Recall what a reference frame is.

And therefore, what is the elapsed round trip time for one tick of Jill's clock, *from Jill's perspective*? *Hint:* Your answers in Step 4 might be useful here.

Check your previous answer with the result reported by the simulation. How much time did Jill think elapsed? Does this match your prediction?

STEP 7: Now let's switch back to Jack's perspective. Recall Jack was using his light clock to time how long it took for Jill's light clock to make one tick. Looking at the results reported by the simulation, what did Jack observe for the time elapsed by one tick of Jill's clock?

Does this result agree with your prediction at the end of Step 5?

STEP 8: Compare your answers in Step 6 (from Jill's perspective) to your answers in Step 7 (from Jack's perspective). Is Jill's light clock running slow, or not? Does the answer depend on who is observing the light clock? How so?

This is one of the surprising conclusions of relativity. If the speed of light truly is the same for all observers, then a clock can appear to tick at a different rate, depending on the relative motion of the clock and an observer… *even if it's the same clock!*

In our example, Jill sees her clock as taking a certain time to tick once. But Jack, while timing Jill's clock with his own clock, thinks Jill's clock took *longer than that* to tick once. Yet they are both looking at Jill's clock!

This effect is called **time dilation**.

STEP 9: Let's see what the effect of time dilation is when the clock is moving even faster. In the simulator window, use the slider to increase Jill's speed to 0.5c. Run the simulation again.

We will now compare what happens to Jill's light clock in two scenarios: one in which Jill is traveling 0.25c and one in which Jill is traveling at 0.5c.

All of these questions are from Jack's perspective, watching Jill's clock move (as depicted by the simulation):

> Compared to when Jill was moving at 0.25c, does the light take a longer or shorter path when Jill is moving at 0.5c?

> Compared to when Jill was moving at 0.25c, does Jack think the speed of the light beam in Jill's clock is different now that Jill is moving at 0.5c? Why or why not? Again, remember Einstein's postulate.

> Based on your two previous answers, which light clock would have a longer round trip time (and therefore take longer to "tick"), the one moving at 0.25c or the one moving at 0.5c? Explain your reasoning.

Based on these observations, as Jill's clock moves faster and faster, would you expect it to tick faster or slower when compared to Jack's clock?

SQ5: Summarize time dilation by circling words to complete the sentence below.

Within a given observer's reference frame, a clock that is (moving / at rest) ticks (faster / slower) than a clock that is (moving / at rest). And, the faster a clock is moving, the (faster / slower) it ticks compared to a clock at rest.

STEP 10: Let's compare Galilean and Einsteinian Relativity for the case of a light clock.

Do A and B agree or disagree on the following quantities? Complete the table below by writing Agree or Disagree in each empty box. In the case of Galilean relativity, you can think of the light clocks as behaving essentially just like the ball being tossed up in the moving car.

Quantity	Galilean	Einsteinian
Distance the light travels		
The time it takes for the light to make a round trip		
The speed of light		

The fact that observers don't agree (as shown in the second box of the last column) on the time it takes for the light clock to make one round trip is called **time dilation**.

The fact that the observers don't agree (as shown in the first box of the last column) on the distance light has traveled is called **length contraction**.

But these two results are necessary in order for the two observers to agree (as shown in the last box of the last column) on the **speed of light**.

Time dilation and length contraction are Einstein's explanation – the "yin and yang" – of how two observers *can always agree on the speed of light* regardless of the motion of the observers. This is the foundation of Einstein's theory of relativity that he first published in 1905.

Part 3: Time Dilation

To summarize so far, if one starts with Einstein's postulate of the constancy of speed of light, then one is *forced* to conclude:

> Two observers in different reference frames (like Jack and Jill) *disagree* on how much time has passed. This can be summarized in several ways:
> - Time is not an absolute quantity.
> - The passage of time depends on the observer's reference frame.
> - The passage of time is *relative.*

This radical idea wasn't immediately accepted in 1905. But since 1905, time dilation has been experimentally verified thousands of times.

One famous experiment was the 1971 experiment by American scientists Hafele and Keating in which atomic clocks were placed on four commercial jet airliners and flown around the world twice.

After the trip, they were compared with atomic clocks that remained on the ground in the laboratory. The clocks that flew around the world were *behind* the clocks that remained on the ground by exactly the amount predicted by Einsteinian relativity!

Today, time dilation is verified every single day in particle accelerators and by particles (called muons) that form in the Earth's atmosphere as a result of cosmic particles colliding with Earth's atmosphere.

Adding Velocities

If a spaceship is moving at 0.5c and it turns on its headlights, the light beam is moving at c, even as measured by an observer outside the spaceship. But 0.5 + 1 is not equal to 1! Isn't there a mathematical contradiction here?

It turns out that velocities don't add in the simple sense (i.e., A + B) we've always expected. Working from his postulate, Einstein developed a new and more accurate (but more complex) equation for adding velocities (i.e., it's not just A + B anymore).

Why don't we notice time dilation in our daily lives?

STEP 1: Let's go back to the simulation and set Jill's speed to the smallest value the simulation allows (0.01c). Now run the simulation. What is the difference in elapsed time for the two clocks (if it's even discernible within the precision of the simulation)?

That small difference arises when Jill is moving at 0.01c. In other words: when Jill is moving at 1% the speed of light.

The speed of light is about 700 million miles per hour. How fast is 1% the speed of light?

Even when we're on a jet airplane traveling at 550 mph, we're moving *less than one millionth* the speed of light.

Applying your answer to SQ5, how do you expect the time dilation effect when you're on an airplane would compare to Jill's time dilation when she was moving at 1% the speed of light?

SQ6: In the Hafele and Keating experiment, why did they need the accuracy of atomic clocks on the airplanes in order to measure time dilation?

Twins and Time Travel

What if humans *did* travel at speeds near the speed of light? This leads to things like the famous twins thought experiment:

Twins Thought Experiment

> Jack and Jill are twins, born at the same time. They are the same age.
>
> Jill gets on a spaceship and travels at 80% the speed of light to our nearest neighboring star system, Alpha Centauri.
>
> Jill returns to Earth at the same speed (total round trip distance is about 8 light years).
>
> When she gets back to Earth, Jack has aged 10 years and Jill has aged only 6 years. The twins are no longer the same age!

Time dilation is not an effect that arises from inaccurate clocks; *it is the very nature of time*. Jack has biologically aged 10 years and Jill has biologically aged 6 years. Literally more time has passed for Jack than for Jill.

But everything else on Earth has also aged 10 years, not just Jack. In other words, Jill has traveled 10 years into Jack's future even though she aged only 6 years. This is time travel!

To take a more extreme example, if Jill travels at 99.99% the speed of light to Alpha Centauri and back, Jack ages about 8 years on Earth but for Jill the trip only takes *52 days*.

So *time travel into the future* is very real and in principle it is very simple: just travel really, really fast! But, in practice, time travel for us humans has the extremely difficult requirement of needing to accelerate a person to nearly the speed of light.

Time travel into the past is not so simple and whether it's even possible or not is still an unsolved problem.

Part 4: The Famous Equation

Another amazing consequence that can be drawn from Einstein's postulate is his most famous formula: $E = mc^2$ (the exact process by which he went from his postulate to this formula is beyond the scope of this Activity).

The famous formula is actually a simplified version of a more general formula, which is

$$E = \gamma mc^2, \text{ where}$$
$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Here, v is the velocity of an object in a given reference frame.

The thought process that led Einstein to E=mc² was a little more complicated than this!

STEP 1: Let's do some basic calculations with this formula. Warning: There be dragons (i.e., a little bit of math) ahead!

What is the value of γ when something is at rest (i.e., when $v = 0$)?

And therefore, what does $E = \gamma mc^2$ simplify to for something at rest?

So, the famous $E = mc^2$ is actually only true for an object at rest (i.e., when measured within an object's own reference frame). Therefore, this energy is called the **rest mass energy**.

Rest Mass Energy

$E = mc^2$ is the energy something possesses even when it is at rest and has no other forms of energy. It is the energy something contains simply by virtue of having mass.

This is a remarkable result, because it says that something has energy by virtue of the fact that it has mass, and vice versa: something has mass by virtue of the fact that it has energy.

Given what you know about the speed of light, is c^2 a large quantity or small quantity?

Therefore, given some "average" amount of mass m, does that mass have a large rest mass energy or a small rest mass energy? *Hint:* Look at $E = mc^2$.

So even a small amount of matter contains a very large amount of energy. We've already seen one very important consequence of this result this semester: During **nuclear fusion**, a small amount of mass is converted into a large amount of energy and this is what *powers the Sun and stars*.

STEP 2: What is the value of γ when something is moving at the speed of light? And therefore, what is the energy of an object that is moving at the speed of light? To answer this, set v equal to ***c*** in the equation for γ above and then use that result in the formula for energy. Hint: you'll need to use the fact that, in the limit, $1/0 \rightarrow \infty$.

SQ7: How much energy is needed to accelerate an object (with nonzero mass) to the speed of light?

This is why the speed of light is the **cosmic speed limit**, *at least for things that have mass*. Only things that are massless (like light) can move at the speed of light. (FYI, in the case of a massless thing like light, the energy formula changes completely.)

Part 5: Annus Mirabilis and Beyond

Einstein began with the postulate that all observers agree that light moves at the speed *c*, regardless of their motion.

This was a postulate that defied common intuition, but he had the courage (like Alice) to pursue the rabbit deep into the rabbit hole and the perseverance to stick with it to its conclusions, unlikely as they seemed.

Some of the conclusions that came from this postulate are:

- The rate of the passage of time is *relative* and not absolute.
- Velocities don't simply add in the way we always expected.
- Energy has mass and vice versa ($E = mc^2$).
- Nothing with mass can achieve the speed of light and therefore the speed of light is the cosmic speed limit.
- Only massless things can travel at the speed of light.
- Time travel into the future is possible in principle.

Using only his faculty of thought (i.e., he did *no experiments*), he was able to draw remarkable conclusions about fundamental aspects of our universe like time, space, mass, and energy. This is an unparalleled triumph of the power of pure thought and the human mind.

His ideas were so revolutionary that, even though he first published relativity in 1905, it took until at least 1919 before the scientific community thought he might be correct.

For all of these reasons, the following quote from Einstein feels especially appropriate and poignant:

"Imagination is more important than knowledge." --Albert Einstein

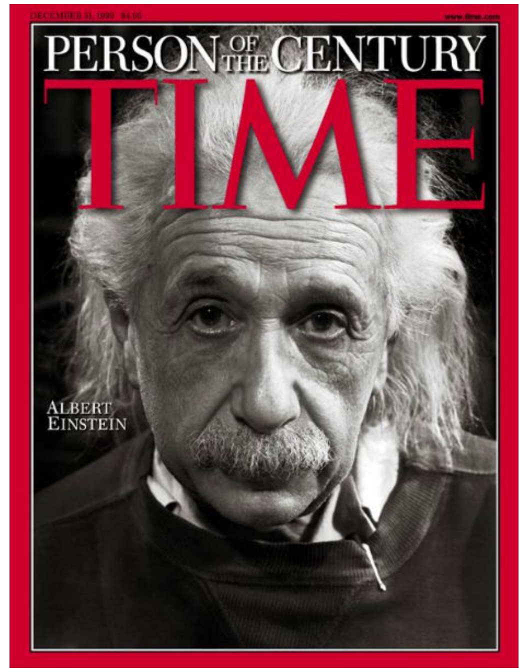
Also in 1905:

- Einstein published his paper on the **photoelectric effect** in which his experiments established that light can behave like a particle (the "particle model of light" that we encountered earlier this semester). This paper earned him the 1921 Nobel Prize in Physics.

- And, he published a third paper introducing the idea of Brownian Motion that also received widespread acclaim.

Therefore, 1905 has been dubbed his "**Annus Mirabilis**" (Latin for "miracle year"). Not a bad year's work for someone that started the year at 25 years old!

We've just scratched the surface here. Einstein went on to many other discoveries.



"Imagination is more important than knowledge."
–Albert Einstein

For example, after 10 more years of hard work he completed his *general theory of relativity* (as mentioned in the introduction) which was yet another revolution, this time in the way we understand gravity. In describing his efforts, Einstein said:

> *"I am now working exclusively on the gravity problem… [O]ne thing is certain — that never in my life have I tormented myself anything like this. Compared to this problem the original relativity theory is child's play."*
>
> --Einstein, ca. 1912

When the first observational evidence came through in 1919 that Einstein's general theory of relativity might be correct, he became an overnight celebrity. The Times of London on Nov. 7, 1919 ran a front page banner headline that read "REVOLUTION IN SCIENCE -- NEW THEORY OF THE UNIVERSE -- NEWTONIAN IDEAS OVERTHROWN". It's not often at all when a scientist makes the headline on the front page. And the bold claim is that Einstein has dethroned Isaac Newton, the inventor of calculus and undisputed king of physics.

Some of the predictions that came out of Einstein's model of gravity and spacetime (i.e., his general theory of relativity) include **black holes**, **gravitational waves**, and even the **Big Bang** itself.

And Einstein's contributions to physics went well beyond just relativity. His discovery of the particle model of light (one of his 1905 accomplishments) laid some of the key ground work for what would eventually become the model of **quantum physics**. And quantum physics ultimately led to technology like lasers, optical storage media (like DVD's), and computers.

It's hard to quantify how much impact this one individual had on our understanding of the Universe, but hopefully you now have some idea why Einstein's image is now a pop culture icon that is instantly recognized by people all around the world as the emblem for genius.