Activity 3: Distance and Size

Materials List:

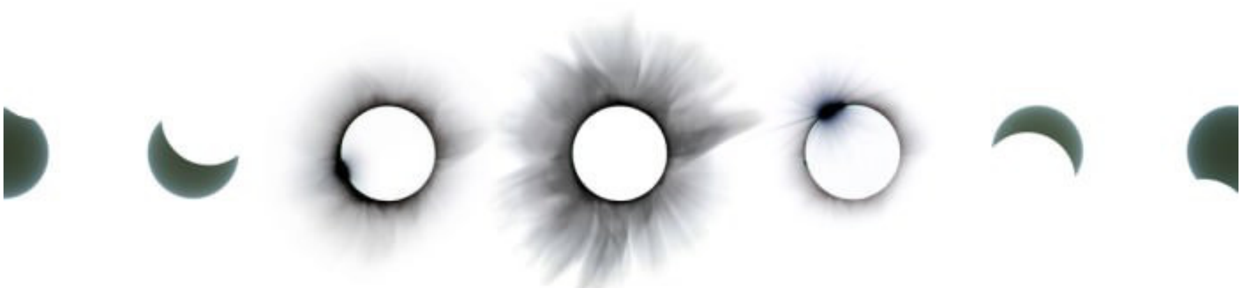
- Scientific Calculator
- Dime
- Scissors
- Roll of Cash Register Paper
- Measuring Tape

Part 1: Angular Size

STEP 1: Look at the photo on the right that was taken while the Moon was low in the sky. Assuming this person is of average height, and given what you know about human height, how large does the Moon *appear* to be? Make an estimate of the Moon's apparent diameter as if the *only* information you had is this photo and the general size of a person.

Is this really the Moon's diameter? If not, then why does it appear to be the size that it does?

STEP 2: During a total eclipse of the Sun, the Moon moves in front of the Sun and appears to be nearly identical in size to the Sun, which is why it can block out the Sun's light "in totality," making it seem like twilight even the middle of the afternoon.



Note: In the above sequence of photos, white and black have been swapped so that the image will print more clearly on white paper.

Do you think the Sun and Moon are really the same size? If not, then why does the Moon appear to be the same size as the Sun?

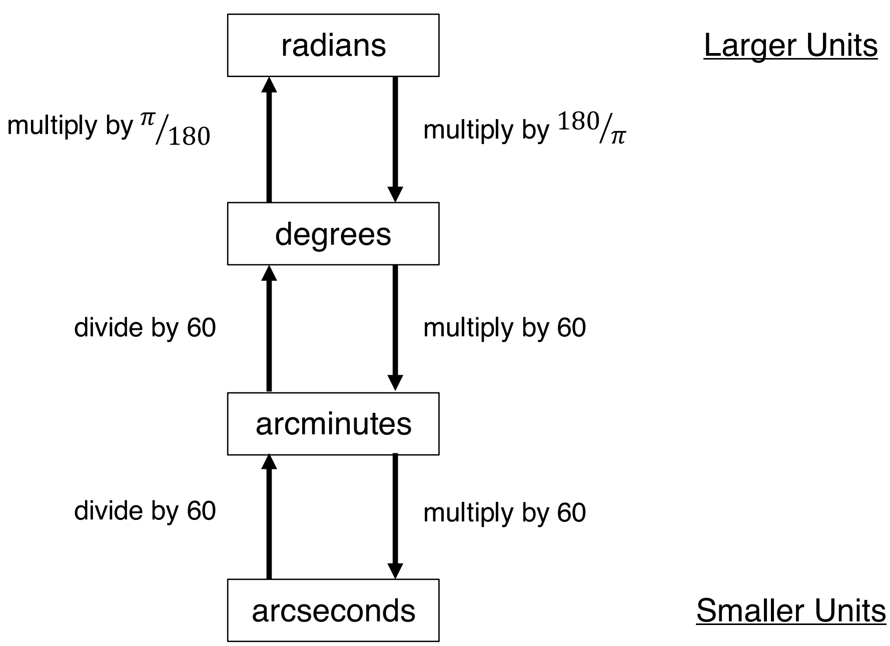
The Moon and Sun during an eclipse *appear* to be the same size, even though they are not. We therefore say that the Moon and the Sun have the same **angular size**.

Angular Size

If you were to point your arms at an object, with one arm pointing to one end of the object and the other arm pointing to the other end, the angular size is the angle that your arms make with one another. In other words, the *angular size* of an object is the *angle that object occupies in your field of view*.

Since the Moon and Sun both occupy the same angle of your field of view, then they have the same angular size.

Angular size is reported in degrees, radians, arcminutes, or arcseconds. There are 2π radians or 360 degrees in a circle. There are 60 arcminutes in a degree, and 60 arcseconds in an arcminute (arcminutes and arcseconds are therefore just like minutes and seconds on a clock, except with the degree replacing the hour). This flowchart will help you convert between these quantities:



Estimating Angular Size

We can get an estimate for the angular size of something in the sky using just our hands. Because human arms and hands are similarly (though not identically) proportioned, we can use them as a crude measuring stick for estimating angular sizes. Here is the procedure:

Step 1: Hold your hand out *at arm's length*.

Step 2: Compare the apparent size of your hand to the apparent size of the object you are looking at, and consult the chart below:

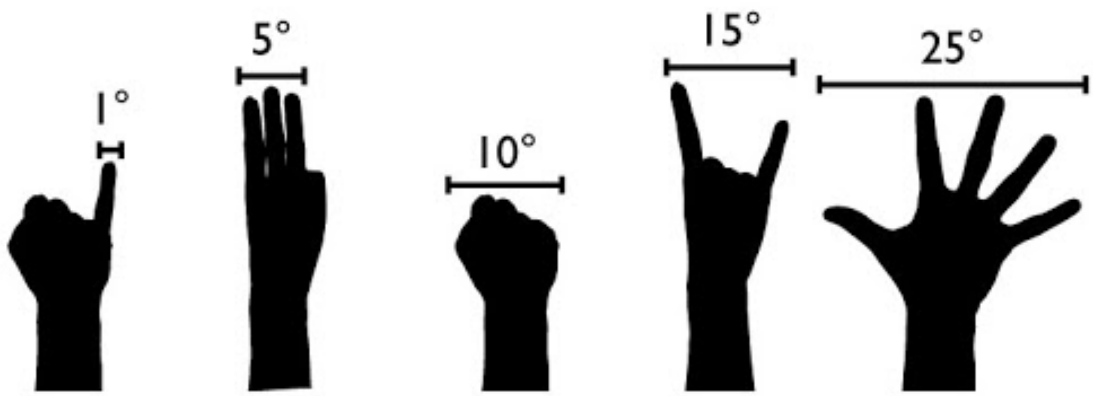


Figure 1 Estimating angular size (image credit: alicesastroinfo.com)

STEP 3: Pick an object that's far away (or at least on the other side of the room) and use the above method to estimate its angular size in degrees.

SQ1:

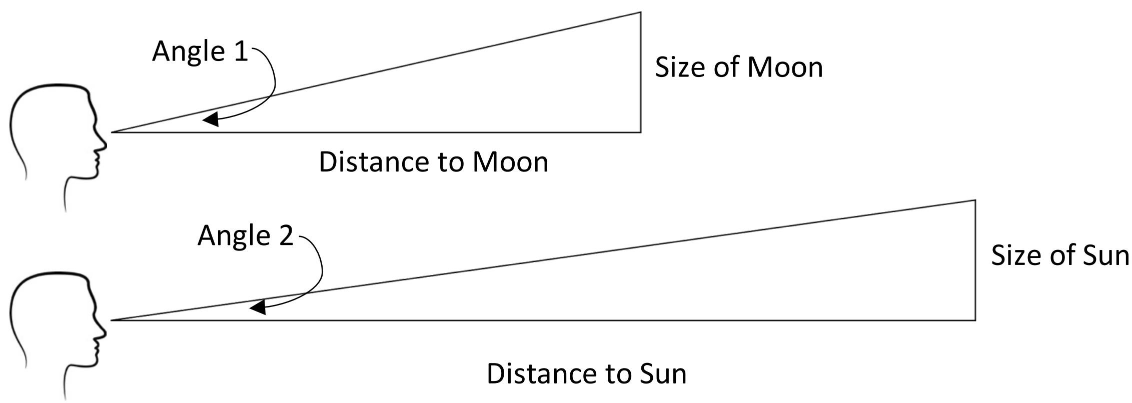
a) The next time you see a Full Moon, you can use the above method to estimate the Moon's angular size. If you do, you'll find that your pinky appears to be roughly twice the width of the Moon. Therefore, what is the approximate angular size of the Moon, in degrees?

b) The true angular size of the Moon is 31.1 arcminutes. Convert this to degrees. How well did the estimate using your hand in part (a) work?

c) If you were to repeat the above procedure with the Sun (which you should definitely NOT do because it would involve looking directly at the Sun), what would you expect to find for the angular size of the Sun? Without even performing the experiment, why do you already know the result?

Part 2: Estimating Distance and Size

STEP 1: Consider the diagram below (not drawn to scale). In each triangle, the short (vertical) side of the triangle corresponds to the *size* of object we're viewing (Moon or Sun) and the long (horizontal) side of the triangle corresponds to the *distance* from your eye to the object we're viewing (Moon or Sun).



The angles on the left side of the triangle are actually the *angular size* of the object being viewed. Therefore:

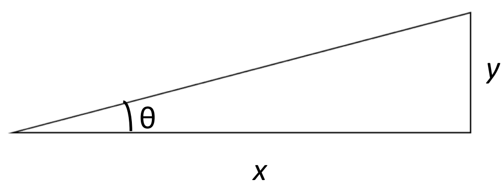
- Angle 1 is the angular size of _________________
- Angle 2 is the angular size of _________________

Based on what you've already seen above, how must Angle 1 relate to Angle 2? Why?

Cotangent Method

Here's a quick review of the trigonometric function called **tangent**. In a right triangle, the tangent of an angle is equal to the length of the side that's *opposite* to the angle divided by the length of the side that's *adjacent* to the angle (you may remember the mnemonic "sohcahtoa" from trigonometry, where "toa" stands for "tangent = opposite / adjacent").

STEP 2: In the right triangle below,



we have the following trigonometric relation:

$$\tan\theta = \frac{y}{x}$$

In the triangle, if *θ* is the *angular size of the Moon*, what do *x* and *y* represent (in words)? Refer back to the triangles on the previous page if you need help.

STEP 3: Using the angular size of the Moon you found in SQ1 along with the tangent function, what is the ratio $\frac{y}{x}$ for the Moon? In other words, you can compute *tan θ* right now, and therefore using the above equality you are also finding $\frac{y}{x}$.
(Note: you'll need to use a scientific calculator to compute the tangent or you can type "*tan ____ deg"* directly into Google.)

$$\tan\theta = \frac{y}{x} =$$

What is $\frac{x}{y}$ for the Moon? (Notice we just took the reciprocal of our previous answer.)

$$\frac{x}{y} =$$

Recalling what *x* and *y* represent, what does the ratio $\frac{x}{y}$ represent for the Moon, in words? Again, refer back to your earlier answers or the triangles on the previous page if you need help.

Cotangent

We just took the tangent of an angle, and then took the reciprocal of the result. This operation is common enough that it has its own name: it's called the **cotangent**. Hence, we are calling this method the "cotangent method".

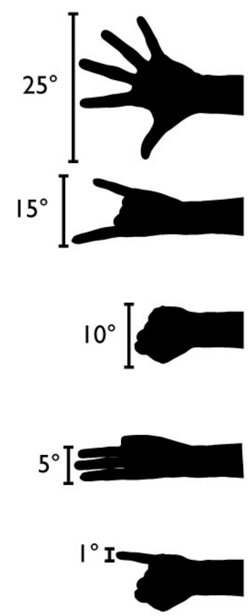
Repeating the steps of the cotangent method for the Sun, what is the ratio $\frac{x}{y}$ for the Sun? Hint: you can skip the math on this one (why?).

SQ2: Using your answers above, fill in the blanks to complete the following sentences.

The distance to the Moon is about _____________ times as large as the Moon's width.

The distance to the Sun is about _____________ times as large as the Sun's width.

<u>Applying the Cotangent Method</u>



STEP 4: Look out of a window that's near you and find an object that is far away (or, if there's no window near you, choose something on the opposite side of the room). Choose an object whose size you can roughly guess, like a car, door, person, window, book, lamp, etc. Hold your arm out straight and use your hand to get an estimate of the angular size of the object.

Helpful tip: As in the diagram on the right, you can also turn your hand on its side if you want to use your hand and finger width to measure the *height* of a distant object.

Your guess for the *actual* size of the object (in units of your choice):

____

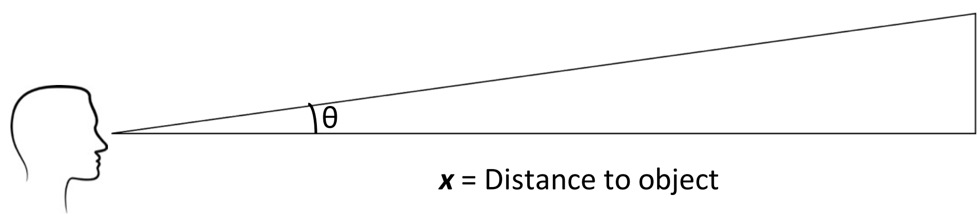
Angular size of object (in degrees, measured using your hands):

____

STEP 5: Now let's use the cotangent method and the triangle below to estimate the distance to the object.

<u>Steps:</u>

1. The long (horizontal) edge of the triangle will be your unknown distance, which we're calling *x*. The short (vertical) edge of the triangle will be the size of the object, which we're calling *y*. Write your estimate for *y* on the triangle below.
2. Now that you have an estimate for the angular size θ of the object using your hand, write your estimate for θ on the triangle below.
3. Calculate tan θ, then take the reciprocal (i.e., find the cotangent of θ).
4. The number you've computed in the previous step is equal to $\frac{x}{y}$. Plug in your guess for *y* and solve for *x*.
5. What are the units for x? Each side of the triangle should always have the *same* units as the other sides. So when you chose units for your estimate of the size of the object, you *also* chose the units for the unknown distance *x*.



What is your estimate for the distance to the object (don't forget to include units)?

$x =$

Remembering that we are just estimating, does your result seem reasonable for the distance from you to the object? Ask your instructor for feedback on your estimate.

What are some potential sources of error that might contribute to your estimate being incorrect?

In summary, just by measuring the angular size of an object, and having a guess about the size of an object, we can use that to figure out how far away the object is. Let's try this in an astronomical context.

<u>Light Years</u>

First, let's define a common unit of distance that's used in astronomy: the *light year*.

> A **light year** is a unit of *distance*. It is the *distance that light travels in one year*. Since light is so fast, it therefore travels a tremendous distance (by human standards) in one year. Specifically, it travels 5.88 *trillion* miles in one year. So a light year is equal to about 6 trillion miles.

If you find it confusing that "light year" (which sounds like a unit of time) is a measure of distance, think of it this way: When someone asks you, "How far is it to Houston from Austin?" you might reply, "Two and a half hours" (depending on what part of Houston you're going to). Of course you don't mean the *distance* is actually two and a half hours, because that's a measure of time. What you mean is, it's the *distance that I can travel in two and a half hours doing the speed limit* on the highway. And the speed of light is, analogously and also literally, the speed limit.

STEP 6: Now let's use our method to estimate an astronomical distance.

SQ3: The only easily visible galaxy to us in the Northern Hemisphere is the **Andromeda Galaxy** (Hubble photo at right). The Andromeda Galaxy contains an estimated *one trillion* stars. The bright nucleus of the galaxy can be seen with the unaided eye on a very clear, moonless night. Astronomers estimate that the nucleus is about 20,000 light-years across. It appears in the sky to be approximately half of your pinky in width. Estimate the distance to the Andromeda Galaxy using the cotangent method.

Estimating Size

STEP 7: So far, we've been finding distances to objects when we already have an estimate of their size. We can also do the opposite and estimate the size of an object if we know something about the distance to it.

In 1716, English astronomer Edmond Halley (of Halley's Comet fame) used a technique called **parallax** to estimate the distance to the Sun. We'll discuss parallax (and do it ourselves) next week. For now, let's just look at his findings. Halley found that the Sun is about *150 million kilometers* away (or about 93 million miles). Given that information, combined with the distance-to-size ratio of the Sun from SQ2, what is the diameter of the Sun in km?

Part 3: Scale Model of the Earth, Moon, and Sun

SQ4: In this Part we will construct a *scale model* of the Earth, Moon, and Sun. What is meant by a *scale model*?

Talk to your instructor about your previous answer before moving on.

STEP 1: An astronaut standing on the Moon would see that the Earth takes up approximately two pinky-finger-widths in the sky. What is the approximate angular size of the Earth as seen from the Moon?

Using your answer above, what is the distance from the Earth to the Moon in *units of Earth diameters*? In other words, what is the ratio of the Earth-Moon distance to the Earth's size? Use the cotangent method.

STEP 2: Given the previous result, can you make a scale drawing of the Earth and the Moon using the grid below? **The Earth has 4 times the diameter of the Moon**. Try to get *both* the *sizes* and the *distances* to scale. Tip: You will want to plan ahead to make sure your drawing will fit on the grid before you start drawing anything. The grid is about 50 squares tall by 70 squares wide.

STEP 3: Now let's build a scale model of the Earth, Moon, and Sun in which the Earth is represented by a dime.

Using your result from Step 1, how far away from the dime "Earth" would our scale model "Moon" need to be? The diameter of a dime is 1.8 cm.

The Earth has about 4 times the diameter of the Moon. How big should our Moon model be?

Can you find an object that has roughly this size? If not, draw a scale version of the Moon on a piece of paper and that will be your model Moon. Now hold the model Moon and the dime Earth next to each other with the appropriate distance between them.

You've created a scale model of the Earth and Moon. Is there anything about this result that surprises you?

SQ5: Look at the following composite photograph of the Earth and Moon. What are the inaccuracies in this depiction as far as scale goes?

STEP 4: Now let's include the Sun in our model.

Before we begin, let's take a guess: if a dime represents the size of the Earth, what every day object might represent the size of the Sun? Don't worry about whether it's right or not, just write down a guess and keep going.

The diameter of the Earth is 12,750 km. In Step 7 of Part 2 we calculated that the diameter of the Sun is about 1.4 million km. How many Earth diameters fit inside one Sun diameter?

Given this, and given that a dime is about 1.8 cm in diameter, what should the diameter of our scale model Sun be? Write your answer in meters, recalling that 1 m = 100 cm.

If there are 3.3 feet in a meter, what does the diameter of our model Sun need to be in feet (rounded to the nearest tenth of a foot)?

Can you find a ball with this diameter? Probably not! Instead, imagine something with the appropriate height to represent the Sun in our model. What is your item?

STEP 5: Compare your dime to your model for the Sun and look at the size difference. This is the relative size of the Earth and Sun!

How far apart should your model Earth and Sun be? Utilizing your answer to SQ2, figure out how far apart we should place the Earth and Sun in our model, in meters.

If a football field is 100 yards, which is about 300 feet or 91.5 meters, how many football fields apart should your Earth and Sun be?

SQ6: Summarize your Earth-Sun model. What is modeling the Earth, what is modeling the Sun, and how far apart are they?

Part 4: Scale Model of the Solar System[1]

> "Space is extremely well named."
> --Bill Bryson, *A Short History of Nearly Everything*

Space is so vast that it can be hard to visualize the distances between things, even within our own Solar System. In this Part, we will construct a scale model of the Solar System to try to wrap our minds around the size of the Solar System.

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[1] This Part adapted from *Voyage through the Solar System* by Dynamic Earth UK

STEP 1: Take your roll of cash register paper and cut off a piece that is 58 ½" in length. Why 58 ½"? Because each inch now represents *100 million kilometers*. This is the *scale* of our scale model: *1" = 100 million kilometers*

STEP 2: At one end of the paper, use your marker to write the word **Sun.** At the other end write **Pluto**. So that the paper doesn't curl, you might wish to tape the ends of the paper to a table or the floor or set something heavy on each end of the paper.

STEP 3: At the halfway point, 28 ¾" from the end marked Sun, draw a line across the paper and label it **Uranus**. This line represents a portion of the arc of the orbit of Uranus.

STEP 4: About halfway between Uranus and Pluto (45" from the Sun), draw another line and label it **Neptune**.

Notice that half of the Solar System is taken up with just two planets (Neptune and Uranus) and the Kuiper belt of asteroids and dwarf planets (which includes Pluto). All of the rest of the Solar System has to fit in the other half!

STEP 5: Draw a line across the paper at 14 ¼" from the Sun and label this one **Saturn**. Another line at 7 ¾" should be labeled **Jupiter**.

This means that the large gas planets occupy ¾ of the Solar System and the inner, rocky planets (Mercury, Venus, Earth, and Mars) must fit in the other ¼!

STEP 6: Using the information in the following table, place markings for the four inner planets plus Ceres (which is in the asteroid belt). Remember, our scale is 1” = 100 million km.

Object	Diameter (in thousands of km)	Diameter (in Earth diameters)	Distance from Sun (in millions of km)	Distance from Sun (in AU)
Sun	1,390.0	109.00	0	0
Mercury	4.9	0.38	58	0.39
Venus	12.1	0.95	108	0.72
Earth	12.8	1.00	150	1
Mars	6.8	0.53	228	1.52
Ceres (in asteroid belt)	0.9	0.07	416	2.77
Jupiter	143.0	11.20	778	5.2
Saturn	120.6	9.40	1427	9.54
Uranus	51.1	4.00	2870	19.2
Neptune	49.5	3.90	4497	30.1
Pluto (inner edge of Kuiper belt)	2.3	0.18	5850	39.5
Eris	2.4	0.19	10200	

STEP 7: Now let’s draw in the planets themselves. How large would they be in this scale model? Let’s start with the Earth. Look at the Earth’s diameter in the above table and recall that our model’s scale is 1” = 100 million km. How big would the Earth be in our model? Describe it in words and/or numbers.

Let's look at the largest planet, Jupiter. How large would it be in our scale model? Again, describe it in words and/or numbers.

Given this information, go ahead and draw in all of the planets to scale (as best you can) on your strip of paper.

STEP 8: The object Eris (in the last line of the table) was discovered in 2005. It was originally called the 10^{th} planet. How does the diameter of Eris compare to the diameter of Pluto?

Dwarf Planet

> Many other objects of a similar size have been found in the outer reaches of the Solar System. It was Eris and other objects like it that led astronomers to create a new category called "**dwarf planet**" and group Pluto, Eris, and other similar objects into that category. **Ceres**, the largest object in the asteroid belt, also fits within the new definition of dwarf planet. Therefore, by introducing the new term, Ceres received a "promotion" from asteroid to dwarf planet.

As of 2020 there have been 5 dwarf planets named (Eris, Pluto, Ceres, Haumea, and Makemake) but there are over 100 more objects that we know of that are suspected to be dwarf planets. Current estimates expect we'll find at least 200 dwarf planets in our Solar System. Most of these are found in a belt of objects (similar to the asteroid belt) that lies beyond Neptune. This belt has been named the **Kuiper Belt**.

Let's remind ourselves of another useful unit of distance, the Astronomical Unit (AU).

> One **Astronomical Unit** (AU) is *defined* to be the distance between the Earth and the Sun. The distance between the Earth and Sun actually varies a bit – because Earth's orbit is an ellipse – so 1 AU is defined to be equal to the *average* distance between the Earth and Sun.

STEP 9: Look at the table above and use a proportion to find the distance between Eris and the Sun in AU. Record this result in the empty bottom right cell.

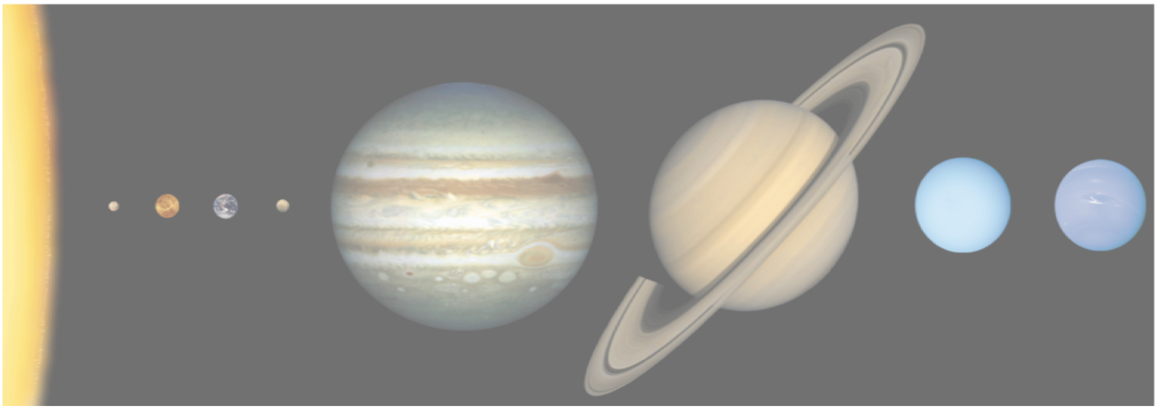
Eris, like Pluto and other objects in the Kuiper belt, have highly eccentric (meaning elliptical) orbits. Therefore, as you saw in the simulation of elliptical orbits in the previous Activity, this means that the distance between Eris and the Sun varies dramatically. The number you calculated is actually the average distance. Currently, Eris is 96.3 AU from the Sun!

Roughly how many times farther away from the Sun is Eris (right now) as compared to Pluto (on average, given in the table)? Where would this place Eris in your paper model of the solar system?

How long would your piece of paper need to be to include Eris?

Interesting Fact: Like most other dwarf planets, Pluto has a very elliptical orbit. Pluto's orbit takes 248 years to make one lap around the Sun. And, for about 20 of those years, Pluto actually passes inside the orbit of Neptune, making Neptune farther from the Sun than Pluto. This last occurred from 1979 to 1999. It won't occur again until approximately the year 2227.

SQ7: Look at the depiction of the Solar System below that was taken from a textbook.



a) What are the inaccuracies in this depiction?

b) Why do you think a textbook would depict the Solar System in this way? To answer this, let's divide it into two questions:

1) If a textbook publisher tried to create and print a scale model like the one you made on the cash register paper, what problem(s) would they run into?

2) What information is contained in the textbook's depiction above that is not contained in your scale model on the cash register paper?